Mathematics: analysis and approaches
Higher level
Paper 2

Topic: Derivatives

Candidate name

35 minutes

Instructions to candidates

- Write your name in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions.
- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[35 marks]**.

8 pages

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 10]

Consider the function $f(x) = \frac{\sqrt{x}}{\sin x}, 0 < x < \pi$.

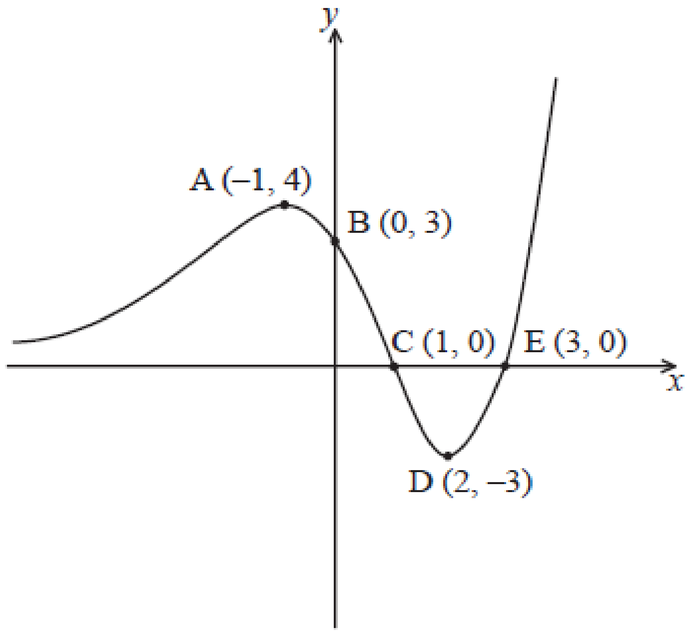
(a) (i) Show that the x-coordinate of the minimum point on the curve $y = f(x)$ satisfies the equation $\tan x = 2x$.

(ii) Determine the values of x for which $f(x)$ is a decreasing function. [7]

(b) Sketch the graph of $y = f(x)$ showing clearly the minimum point and any asymptotic behaviour. [3]

2. [Maximum mark: 4]

The graph of $f(x)$ is shown below, where A is a local maximum point and D is a local minimum point.



Sketch the graph of the derivative $f'(x)$, clearly showing the coordinates of the images of the points A and D, labelling them A'' and D'' respectively.

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

3. [Maximum mark: 21]

A curve C has the equation $y = \frac{2x^2+6x-3}{x+k}, x \in \mathbb{R}, x \neq -k$, where k is a real positive constant.

(a) Show that $\frac{dy}{dx} = \frac{2x^2+4kx+6k+3}{(x+k)^2}$. [5]

(b) Find the range of values of k for which a local minimum or maximum point exists. [4]

Consider the curve C, when k = 2.

(c) Write down the equation of the vertical asymptote. [1]

(d) Find the equation of the oblique asymptote. [4]

(e) Show that $\frac{dy}{dx} > 2$, for $x \in \mathbb{R}, x \neq 2$. [4]

(f) Sketch the curve C, showing clearly both asymptotes and the general behaviour of C as it approaches each asymptote. [You are not required to find any axes intercepted.] [4]